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MOMENTS OF INERTIA.

By G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

It is the purpose of this paper to put on record formulæ for the Moments of Inertia of the plane areas, $\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} = 1$, and the solid bounded by the surface, $\left(\frac{x}{a}\right)^{\frac{2}{2m+1}} + \left(\frac{y}{b}\right)^{\frac{2}{2n+1}} + \left(\frac{z}{c}\right)^{\frac{2}{2p+1}} = 1$.

Let μ be the mass of a unit, (a) area, (b) volume.

(a) Areas, when n and m are positive integers.

For the x -axis,

$$I = 4\mu \iint y^2 dx dy = \frac{4ab^3\mu}{4} \cdot \frac{\Gamma(m+\frac{1}{2})\Gamma(3n+\frac{3}{2})}{\Gamma(m+3n+3)} \\ = \frac{\mu ab^3(2m+1)(2n+1)(6n+1)\Gamma(m+\frac{1}{2})\Gamma(3n+\frac{1}{2})}{2(m+3n+2)(m+3n)(m+3n+1)\Gamma(m+3n)} \dots\dots\dots (1)$$

$$= \frac{1.3.5\dots(2m+1) \times 1.3.5\dots(6n+1)}{2 \cdot 4.6\dots 2(m+3n+2)} \cdot 2\pi\mu ab^3(2n+1) \dots\dots (2).$$

For the y -axis,

$$I_1 = 4\mu \iint x^2 dx dy = \frac{\mu a^3 b(2m+1)(2n+1)(6m+1)\Gamma(3m+\frac{1}{2})\Gamma(n+\frac{1}{2})}{2(3m+n+2)(3m+n+1)(3m+n)\Gamma(3m+n)} \dots\dots\dots (3)$$

$$= \frac{1.3.5\dots(6m+1) \times 1.3.5\dots(2n+1)}{2 \cdot 4.6\dots 2(3m+n+2)} \cdot 2\pi\mu a^3 b(2m+1) \dots\dots (4).$$

For an axis through its center perpendicular to its plane,

$$I_2 = I + I_1 \dots\dots\dots (5).$$

The product of inertia of a quadrant about its axes is,

$$p = \mu \iint xy dx dy = \frac{\mu a^2 b^2}{4} \cdot \frac{\Gamma(2m+2)\Gamma(2n+1)}{\Gamma(2m+2n+3)} \\ = \frac{\mu a^2 b^2 mn(2m+1)(2n+1)\Gamma(2m)\Gamma(2n)}{4(m+n+1)(m+n)(2m+2n+1)\Gamma(2m+2n)} \dots\dots\dots (6)$$

$$\frac{1.2.3.4.\dots(2m+1)\times 1.2.3.4.\dots(2n+1)}{1.2.3.4.\dots(2m+2n+2)} \cdot \frac{\mu a^2 b^2}{4} \dots\dots\dots(7).$$

Let $m=n=0$. Then for the ellipse, $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$.

$$I = \frac{1}{4} \pi \mu a b^3, \quad I_1 = \frac{1}{4} \pi \mu a^3 b, \quad I_2 = \frac{1}{4} \pi \mu a b (a^2 + b^2), \quad p = \frac{1}{8} \mu a^2 b^2.$$

Let $m=n=1$. Then for the hypocycloid, $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$.

$$I = \frac{2}{5} \frac{1}{3} \pi \mu a b^3, \quad I_1 = \frac{2}{5} \frac{1}{3} \pi \mu a^3 b, \quad I_2 = \frac{2}{5} \frac{1}{3} \pi \mu a b (a^2 + b^2), \quad p = \frac{1}{8} \mu a^2 b^2.$$

(b) Solids, when m , n , and p are positive integers.

With regard to the plane (yz),

$$\mathbf{I} = 8\mu \iiint x^2 dx dy dz$$

$$\frac{\frac{8\mu a^3 bc}{8}}{(2m+1)(2n+1)(2p+1)} \cdot \frac{\frac{I'(3m+\frac{3}{2})I'(n+\frac{1}{2})I'(p+\frac{1}{2})}{I'(3m+n+p+\frac{3}{2})}}{\dots\dots\dots}$$

$$\frac{4\mu a^3 bc(2m+1)(2n+1)(2p+1)(6m+1)I'(3m+\frac{1}{2})I'(n+\frac{1}{2})I'(p+\frac{1}{2})}{(6m+2n+2p+5)(6m+2n+2p+3)(6m+2n+2p+1)I'(3m+n+p+\frac{1}{2})} \dots\dots(8)$$

$$\frac{1.3.5.\dots(6m+1)\times 1.3.5.\dots(2n+1)\times 1.3.5.\dots(2p+1)}{1.3.5.\dots(6m+2n+2p+5)} \times 4\mu \pi a^3 bc(2m+1) \dots\dots(9).$$

With regard to the plane (xz),

$$\mathbf{I}_1 = 8\mu \iiint y^2 dx dy dz$$

$$\frac{4\mu a b^3 c(2m+1)(2n+1)(2p+1)(6n+1)I'(m+\frac{1}{2})I'(3n+\frac{1}{2})I'(p+\frac{1}{2})}{(2m+6n+2p+5)(2m+6n+2p+3)(2m+6n+2p+1)I'(m+3n+p+\frac{1}{2})} \dots\dots(10)$$

$$\frac{1.3.5.\dots(2m+1)\times 1.3.5.\dots(6n+1)\times 1.3.5.\dots(2p+1)}{1.3.5.\dots(2m+6n+2p+5)} \times 4\mu \pi a b^3 c(2n+1) \dots\dots(11).$$

With regard to the plane (xy),

$$\mathbf{I}_2 = 8\mu \iiint z^2 dx dy dz$$

$$\frac{4\mu a b c^3(2m+1)(2n+1)(2p+1)(6p+1)I'(m+\frac{1}{2})I'(n+\frac{1}{2})I'(3p+\frac{1}{2})}{(2m+2n+6p+5)(2m+2n+6p+3)(2m+2n+6p+1)I'(m+n+3p+\frac{1}{2})} \dots\dots(12)$$

$$= \frac{1.3.5 \dots (2m+1) \times 1.3.5 \dots (2n+1) \times 1.3.5 \dots (6p+1)}{1.3.5 \dots (2m+2n+6p+5)} \times 4\mu\pi abc^3(2p+1) \dots (13).$$

$$\mathbf{I}_3 = \mathbf{I} + \mathbf{I}_1, \text{ for } z\text{-axis, } \mathbf{I}_4 = \mathbf{I} + \mathbf{I}_2, \text{ for } y\text{-axis,}$$

$$\mathbf{I}_5 = \mathbf{I}_1 + \mathbf{I}_2, \text{ for } x\text{-axis, } \mathbf{I}_6 = \mathbf{I} + \mathbf{I}_1 + \mathbf{I}_2, \text{ for center.}$$

Product of inertia of an octant of the solid with regard to the (y, z) axes,

$$P = \mu \iiint yz dx dy dz = \frac{\mu ab^2 c^2}{8} \cdot \frac{I'(m + \frac{1}{2})I'(n+1)I'(2p+1)}{I'(m+2n+2p+\frac{1}{2})} \cdot \frac{4\mu ab^2 c^2 np(2m+1)(2n+1)(2p+1)I'(m+\frac{1}{2})I'(2n)I'(2p)}{(2m+4n+4p+5)(2m+4n+4p+3)(2m+4n+4p+1)I'(m+2n+2p+\frac{1}{2})} \dots (14)$$

$$= \frac{1.2.3 \dots (2n+1) \times 1.2.3 \dots (2p+1) \times \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \dots \left(\frac{2m+1}{2} \right)}{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \dots \left(\frac{2m+4n+4p+5}{2} \right)} \cdot \frac{\mu ab^2 c^2}{4} \cdot \frac{1.2.3.4 \dots (2n+1) \times 1.2.3.4 \dots (2p+1)}{(2m+3)(2m+5) \dots (2m+4n+4p+5)} \cdot \mu ab^2 c^2 \cdot 2^{2(n+p)} \dots (15).$$

With regard to the axes (x, z) ,

$$P_1 = \mu \iiint xz dx dy dz = \frac{4\mu a^2 bc^2 mp(2m+1)(2n+1)(2p+1)I'(2m)I'(n+\frac{1}{2})I'(2p)}{(4m+2n+4p+5)(4m+2n+4p+3)(4m+2n+4p+1)I'(2m+n+2p+\frac{1}{2})} \dots (16)$$

$$= \frac{2^{2(m+p)} 1.2.3.4 \dots (2m+1) \times 1.2.3.4 \dots (2p+1)}{(2n+3)(2n+5) \dots (4m+2n+4p+5)} \cdot \mu a^2 bc^2 \dots (17).$$

With regard to the axes (x, y) ,

$$P_2 = \mu \iiint xy dx dy dz = \frac{4\mu a^2 b^2 cmn(2m+1)(2n+1)(2p+1)I'(2m)I'(2n)I'(p+\frac{1}{2})}{(4m+4n+2p+5)(4m+4n+2p+3)(4m+4n+2p+1)I'(2m+2n+p+\frac{1}{2})} \dots (18)$$

$$= \frac{2^{2(m+n)} 1.2.3.4 \dots (2m+1) \times 1.2.3.4 \dots (2n+1)}{(2p+3)(2p+5) \dots (4m+4n+2p+5)} \cdot \mu a^2 b^2 c \dots (19).$$

Let $m=n=p=0$. Then for $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$,

$$\mathbf{I} = \frac{4}{15} \mu \pi a^3 bc, \quad \mathbf{I}_1 = \frac{4}{15} \mu \pi ab^3 c, \quad \mathbf{I}_2 = \frac{4}{15} \mu \pi abc^3,$$

$$\mathbf{I}_3 = \frac{4}{15} \mu \pi abc(a^2 + b^2), \quad \mathbf{I}_4 = \frac{4}{15} \mu \pi abc(a^2 + c^2),$$

$$\mathbf{I}_5 = \frac{4}{15} \mu \pi abc(b^2 + c^2), \quad \mathbf{I}_6 = \frac{4}{15} \mu \pi abc(a^2 + b^2 + c^2),$$

$$P = \frac{4}{15} \mu ab^2 c^2, \quad P_1 = \frac{4}{15} \mu a^2 bc^2, \quad P_2 = \frac{4}{15} \mu a^2 b^2 c.$$

Let $m=n=p=1$. Then for $\left(\frac{x}{a}\right)^{\frac{3}{2}} + \left(\frac{y}{b}\right)^{\frac{3}{2}} + \left(\frac{z}{c}\right)^{\frac{3}{2}} = 1$,

$$\mathbf{I} = \frac{4}{715} \mu \pi a^3 bc, \quad \mathbf{I}_1 = \frac{4}{715} \mu \pi ab^3 c, \quad \mathbf{I}_2 = \frac{4}{715} \mu \pi abc^3,$$

$$\mathbf{I}_3 = \frac{4}{715} \mu \pi abc(a^2 + b^2), \quad \mathbf{I}_4 = \frac{4}{715} \mu \pi abc(a^2 + c^2),$$

$$\mathbf{I}_5 = \frac{4}{715} \mu \pi abc(b^2 + c^2), \quad \mathbf{I}_6 = \frac{4}{715} \mu \pi abc(a^2 + b^2 + c^2),$$

$$P = \frac{64 \mu ab^2 c^2}{15.13.11.7.5}, \quad P_1 = \frac{64 \mu a^2 bc^2}{15.13.11.7.5}, \quad P_2 = \frac{64 \mu a^2 b^2 c}{15.13.11.7.5}.$$

Thus we could multiply examples without number.

Formulæ (1), (3), (6), (8), (10), (12), (14), (16), (18), will hold for m, n, p fractional as well as integral.

For the radius of gyration we have

$$k_n^2 = \frac{I_n}{\mu A}, \quad K_n^2 = \frac{\mathbf{I}_n}{\mu V},$$

where A and V are known, (see AMERICAN MATHEMATICAL MONTHLY, page 380, Vol. I., No. 11.)

A SIMPLE DEDUCTION OF THE DIFFERENTIAL OF $\log x$.

By J. W. NICHOLSON, A. M., LL. D., Professor of Mathematics in Louisiana State University.

Let $f(x) = \log x$ (1), then $f(xy) = f(x) + f(y)$ (2).

Differentiate, $f'(xy)(ydx + xdy) = f'(x)dx + f'(y)dy$ (3).

Since (3) is true when x and y are independent,

$f'(xy)ydx = f'(x)dx$ (4), and $f'(xy)xdy = f'(y)dy$ (5).

$$(4) \div (5), \quad \frac{f'(x)}{f'(y)} = \frac{y}{x} = \frac{1/x}{1/y} \quad \text{. (6).}$$

$$\therefore f'(x) = \frac{m}{x}, \quad f'(y) = \frac{m}{y} \quad \text{. (7).} \quad \therefore d \log x = \frac{m}{x} dx \quad \text{. (8).}$$